FUZZY SYLLOGISMS, NUMERICAL SQUARE, TRIANGLE OF CONTRARIES, INTER-BIVALENCE- WITH AN HISTORICAL APPENDIX ON THE QUANTIFICATION OF PREDICATE.

by **Cavaliere Ferdinando** * and **Donnarumma Antonio** **

*Doctor in Philosophy (Bologna University), v. della Libertà 5, 47042 Cesenatico (FC), Italy, t. +39 0547-672469 cavaliere.ferdinando@gmail.com
** Lecturer in Bologna University, Engineering Faculty, v. del Borgo di S. Pietro 123, 40100 Bologna, Italy, t. +39 051-241155 antonio.donnarumma@mail.ing.unibo.it

Abstract

New unpublished **Syllogisms** and **Polysyllogisms**, called "**Distinctivi**" D, take place by means of the **Hexagon** of oppositions, in which the Universal **Uba** (**all or no b are a**) is the contradictoria of the Particular **Yba** (**only some b are a**). Y is preferred, as a *primitive*, to I or O, because it is more "natural" than the others. Typical inferences of the systems are the **obversions: Uba = Uba'**, **Yba = Yba'**. D-systems enclose traditional Syllogisms. Polygonal and Numerical developments are considered, including intermediate quantifiers ("the majority of", …) and some applications in *Modality*, *Semiotic* (synonymous-antonymous), *Propositional Logic* (**semi-implication**). Polygons are finally absorbed into the **Numerical D-Square** NDs. Isomorphisms are discovered between bivalent D-systems and **Non-Standard Logics** by ***depriving*** *the subject-class of the* ***quantifier*** *and* ***transferring*** *its (pre)numerical attribute* ***to the value of truth*** *of the judgement*. These "(Poli-)Inter-bivalent" Logics admit **intermediate values** *between true and false*, as well as a **weakened Principle of Contradiction**. In that way we get Fuzzy Logic, and present the **Fuzzy Cube of Opposition.**

Part 1 PRE-NUMERICAL DISTINCTIVE SYLLOGISMS D

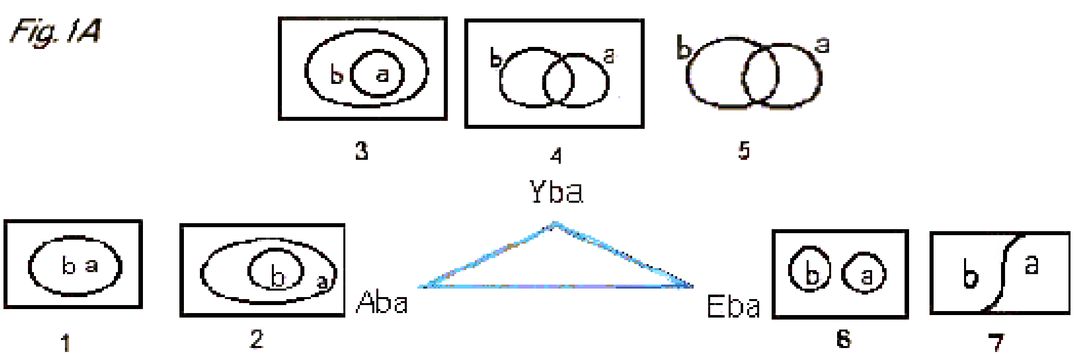
Simple Distinctive Syllogisms

Categorical Triangular Syllogism D3

Among the **Distinctive Syllogisms** presented here (in Latin: "Distinctivi"), the most primitive are the "**Triangular**" ones based on the "triangle" of contraries, which presents the following 3 categorical propositions: universal affirmative, universal negative, and, rather than the 2 Particulars of the classic *oppositive square*, their logical conjunction, which is called **Particular Distinctive** (see figure 1A). The quantifier in the latter can be interpreted in natural language with expressions like **"only some"** in the sense of **excluding** universality, "at least one but not all", "all the…except some", "neither all, nor none", "only a part (among all the…)", etc.

Given an ordered pair of *classes* – one "b" subject class and one "a" predicate class, we may symbolize the 3 categoricals mentioned above in the following way: Aba (= every b is a); Eba (= no b is a); **Yba** (= **only some b is a**). The terminological choice of "Distinctive" (in contrast to Affirmative and Negative) is motivated by the need to place, next to judgments compatible with the totality of a whole, those judgments that produce a "distinction" (*partition*) between parts of a whole, affirming some of these parts in the negation of the rest. [N.b. : for the purposes of this study we will utilize the terms "class" and "set" as synonyms]

Fig. 1A



Near each categorical proposition in Figure 1A, we can see the diagrams that, if taken separately, represent its possible interpretations according to set theory. In these diagrams the *boundary rectangle* represents the *Universe of the discourse*, UD, except in case 5, where the union of b and a covers the entire Universe. (Examples: case 4. : UD = quadrilaterals, b = rhombuses, a = rectangles 5. UD = polygons b= polygons with fewer than five sides a = polygons with more than three sides). The **7 diagrams** are **exhaustive** of the relations between two pairs of complementary sets that are all *different from the Universal class and the Empty class* (see Bird, O. 1964: chapter 3 paragraph 27). The "triangle" of opposition results in a **partition** of these **7 cases**, a circumstance that *does not occur in the oppositive square*. We can therefore propose the Axiom **(Aba) V (Eba) V (Yba)**, where V = aut. Laws of Immediate inference: **a=a''** (double negative); **Aba=Aa'b'** (contraposition); **Eba=Eab** (conversion); **Aba=Eba'** (obversion); **Eba = Aba'** (obversion); **Yba = Yba'** (obversion). The latter may also be proven *within traditional Syllogistics (S)* as follows: Yba = def: Iba *Oba

Iba * Oba = (Eba)' * (Aba)' = (Aba')' * (Eba')' = Oba' * Iba' = def: Yba'

For example: *Only some* scientists are logical if, and only if, only some scientists are not logical.

The **obversion** of the **Particular Distinctive**, **with affirmative copula or predicates**, results in, ***with no change in the quality of the quantifier*** (as instead occurs for the 4 traditional ones) **an equivalent Particular Distinctive, with negative copula or predicate**.

According to these three categorical propositions, it is possible to construct a **syllogistics** that we will call "**triangular"** or **D3,** based on the classical model. Among the 108 possible moods, the **6 valid ones** will be:

1st AAA (**Barbara**), EAE (**Celarent**), **2nd**: EAE (**Cesare**) AEE (**Camestres**); **3rd**: **YAY** (***HydraLynx),*** *new mood*), **4th**: AEE (**Camenes**). The YAY in 3rd figure may be taken as an axiom, but here we intend to demonstrate how it can be derived from traditional syllogistics.

Thesis (Yba * Abc) → Yca

1. (Iba * Abc) → (Ica) (Disamis)

2. (Oba* Abc) → (Oca) (Bocardo)

(1.* 2.) → Ica * Oca (Propositional Logic)

(1.* 2.) → Yca (Definition Yca = Ica * Oca)

[(Iba * Abc) * (Oba * Abc)] → Yca

1. 2.

[(Iba * Oba) * Abc] → Yca (Distributive property of the logical product)

[Yba * Abc] → Yca (Definition Yba) *q.e.d.*

For example: (Only some **s**yllogisms are **A**ristotelian)* (Every **s**yllogism is reasoning) → *Only some* reasoning is not **A**ristotelian) (**Ysa * Asr → Yra'** substituting Yra with Yra').

Exclusive Triangular Syllogism D3x

In Scholastic Logic, the "**only b is/are a**" (= "Tantum b est/sunt a ") type predications were called "Exclusivae". In light of modern syllogistic interpretations they are = **Ab'a'** , or in our symbolism =**Xba.** Xba means that "b 's alone are a 's" or "no a is not b " (while "there are b 's that are not a" is left undefined).. Xba' means: only b is not a. Therefore Xba = Ab'a' = Aab = Xa'b', while Xba' = Aa'b = Ab'a = Xab'. Oppositive squares were derived even from Exclusive propositions (Figure 2). As for the particulars, we can once again generate an oppositive triangle (Figure 3): Xba = only b is a; Wba = every complement of b is a =every not b ("b ad infinitum") is a; Jba = only some complement of b is a (only some not b is a).

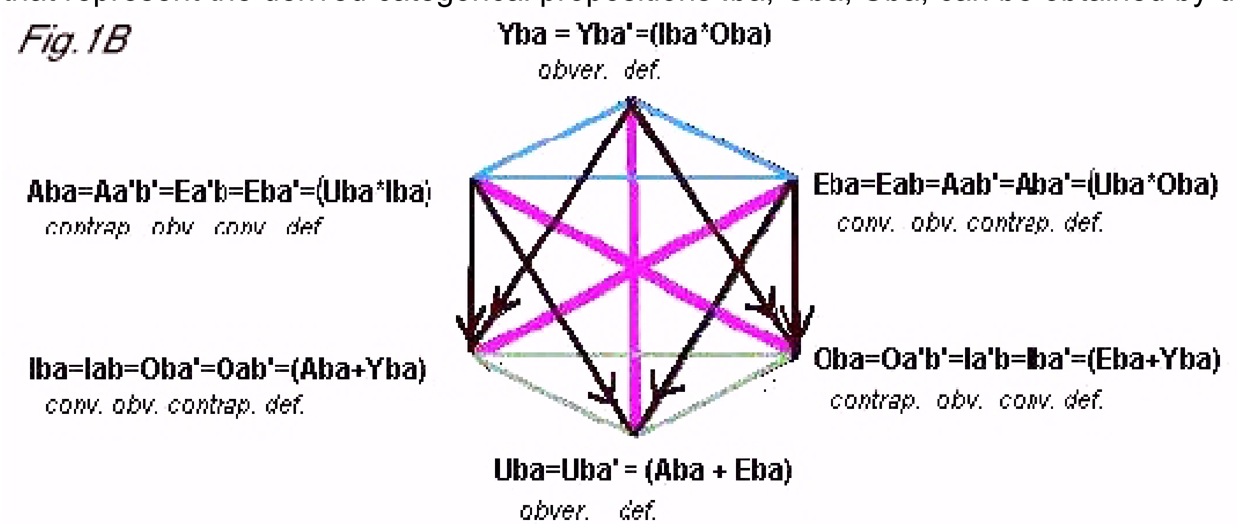
Fig.2 **Xba ------ Xba'**
I X I
(Xba')'----- (Xba)'

Fig.3 **Jba**
**/ **
Xba----- Wba

For these predications, only rules of inference and axioms that are entirely analogous to those of the categoricals will be applied. It therefore gives rise to a distinctive syllogistics with the due replacements of the negative terms as required.

Hexagonal Syllogism D6

At this point, triangular Categorical Propositions will be integrated with traditional Categorical Propositions enriched with negative terms into a Hexagonal Syllogism D6. From the axiom (Aba $\underline{v}$ Eba $\underline{v}$ Yba), we may derive the contradictions, or the negations, of the basic categorical propositions. Therefore, (Aba)‘ $\leftrightarrow$ (Yba $\underline{v}$ Eba) ; (Eba)‘ $\leftrightarrow$ (Aba $\underline{v}$ Yba) ; (Yba)’$\leftrightarrow$(Aba $\underline{v}$ Eba). We recognise the traditional Iba = def. (Aba $\underline{v}$ Yba) and Oba = def. (Eba $\underline{v}$ Yba), while a new categorical proposition appears that we shall call **Universal** “**Distinctive”**, which expresses the **negation** of the **Particular Distinctive** and is symbolized as **Uba**=def. (Aba $\underline{v}$ Eba) (**or every or no b is a**) from the modified formula: “**A**df**I**rmo, n**E**g**O**, in h**Y**bridis disting**U**o”. The obversion is also valid for the U-proposition: Uba = Uba’. Since Eba = Eab then (Eba)’ = (Eab)’; furthermore (Eba)’ = (Aba $\underline{v}$ Yba), as (Eab)’ = (Aab $\underline{v}$ Yab), therefore (Aab $\underline{v}$ Yab) = (Aba $\underline{v}$ Yba), i.e. Iab=Iba. In the resulting oppositive hexagons (see Figure 1B), the <u>contradictory categorical propositions</u> are arranged symmetrically around the **centre**, while the <u>contraries</u> (or <u>sub-contraries</u>) are arranged symmetrically around the **vertical axis.** An ideal horizontal axis separates the primitives, located in the upper part of the hexagon, from the derived categorical proposition below. The possible sets that represent the derived categorical propositions Iba, Oba, Uba, can be obtained by disjunction.



Thus, a **general law** of the syllogism is identified. This law states that <u>for each categorical proposition, the **same law of immediate inference** is valid for its **contradictory** categorical proposition</u> : contraposition for A-O propositions, conversion for E-I propositions, obversion for Y-U propositions.

R. Blanché (1966) and A. Sesmat (1951) already presented oppositive hexagons, but we have found no evidence of syllogistics having ever been developed. Since 1910 N. A. Vasiliev (1925) developed a “triangular” syllogistic based on the “only some” quantifier, but different from ours in that they are similar to paraconsistent systems.

By adding the two cases of obversion to the rules of immediate inference at hand and then systematically extending all of the transformations of equivalence to an unordered pair of terms taken as both negative and positive, 16 basic propositions with different meanings are obtained, as indicated in Table 1. In the last three lines there are expressions not included in the initial hexagon, but rather generated from another 3 hexagons constructed on the pairs b’a’, ab, a’b’(the remaning 4 possible hexagons [or triangles] are equivalent to the others by obversions).

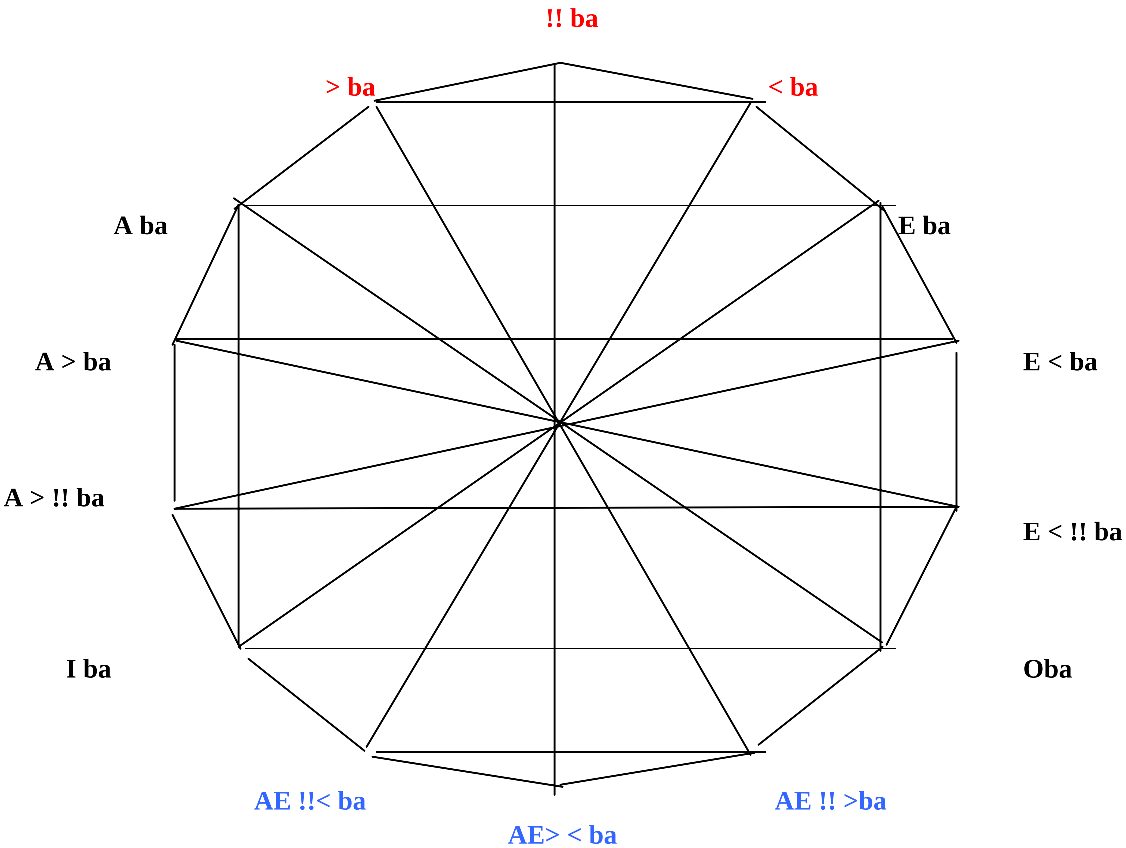
Table 1

Aba=Aa'b'=Ea'b=Eba'	Contradictory of	Oba=Oa'b'=Iba'=Ia'b
Eba=Eab=Aab'=Aba'	"	Iba=Iab=Oba'=Oab'
Yba=Yba'	"	Uba=Uba'
Ab'a'=Aab=Eab'=Eb'a	"	Ob'a'=Oab=Iab'=Ib'a
Eb'a'=Ea'b'=Aa'b=Ab'a	"	Ib'a'=Ia'b'=Oa'b=Ob'a
Yb'a'=Yb'a	"	Ub'a'=Ub'a
Yab=Yab'	"	Uab=Uab'
Ya'b'=Ya'b	"	Ua'b'=Ua'b

The table in Appendix 1 has been divided into two parts, "a" and "b", to facilitate printing. The part "b" should ideally be placed to the right of the "a". It summarizes **all the syllogisms** admissible in the hexagonal system, with the headings of the columns and lines that function respectively as I and II premises, while each proposition inside the cells represents the "stricter" or "stronger" conclusion of the syllogism (subordinate moods are implied). By carrying out the necessary transformations (inverting the order of the premises, conversions, etc.), the table **encompasses** all valid moods of the traditional syllogism (24 moods) and of the syllogism with negative terms (blue conclusions), as well as adding **new syllogistic forms.** The new moods include: in **1st** Figure, **Uma *Acm → Uca** (*UnaLux),* demonstrable by means of indirect reduction from *HydraLynx*; in **2nd** Figure, **Uam * Ycm → Oca** *(VultGyro)*. Moreover, **other deductive forms** appear that are not strictly syllogistic, but are nonetheless valid (and derivable), like: **Uma * Ymc → Yca * Yc'a'**.
The valid syllogisms that include a premise in Y are: 1st **AYI, EYO**; 2nd **AYO, EYO**; 3rd **AYI, YAY, YAI, YAO, EYO**; 4th **YAI, EYO**. The logic of propositions allows us to derive the subordinate moods YAI and YAO (Y→ I, Y→O) from the *HydraLynx* axiom (**Yma * Amc → Yca** in 3rd Figure). The first of these, when the conclusion is converted and the premises switched, leads to AYI; the latter, by obversion of the conclusion and of the first premise, results in the EYO mood. If the first premise is converted, is it possible to reach EYO in 4th Figure from EYO in 3rd Figure; this mood may result in YAI, always in 4th Figure, by means of indirect reduction; from here, conversion of the conclusion and the exchanging of premises results in AYI in 1st Figure, which by obversion of conclusion and the first premise generates EYO, which by conversion of the first premise generates EYO in 2nd Figure. AYO in 2nd Figure is generated from AYI in 1st Figure as follows: contraposition of the first premise and obversion of second premise and conclusion.

Polygonal Syllogisms – Quasi-numerical Syllogisms QD

The A-E side is broken with the Y-proposition or, if preferred, its hiatus is unambiguously closed. At this point we can conceive further interruptions of the segment that are intermediary but not necessarily equidistant. These new quantifiers, and corresponding predications, if chosen in such as way as to be incompatible and exhaustive amongst themselves, may give rise to new **Oppositive Polygons**, or bases for the construction of **Polygonal Distinctive Syllogisms**. The Figure 4 below illustrates the oppositive relations among 5 quantifiers: the traditional A and E, and another three that are called Quasi-numerical, Qn, which we symbolise with >, !! , <, which respectively stand for **only a part that is majority** of**;** the **exact half** of**; only a part that is minority** of. The Qn informs us of its superiority or inferiority or coincidence with an ideal half (not calculated) of the subject elements, as well as its diversity from A and E. From among the 5 quantifiers, therefore, a simple relation of order rules *(in Latin: comparo)*. In the figure there are intermediate and ordered expressions between A and I on the left sides, between E and O on the right sides; Iba, is equivalent to A>!!< ba (every, or only a majority, or exact half, or only a minority part of b is a) and Oba, a E < !! > ba (none or only a minority, or exact half, or only a majority part of b is a). The expressions at the opposite vertices are reciprocally contradictory. The expressions that may be obtained through obversion with exchange of the quantifiers are arranged symmetrically around the central vertical axis. For example: >ba ←→ <ba’ as <ba ←→ >ba’, or A > !! ba ←→ E < !! ba’ . !! is reciprocal to itself, like Y, that is equivalent to > !! <.   Fig.4:



By simplifying the polygon as illustrated in Figure 5, its logical properties remain unchanged even if we compress the sides, reducing the vertices to the 4 canonicals while the others become points inside the perimeter of the classical Square.

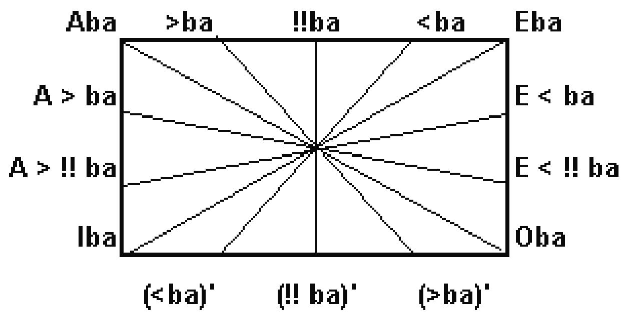


Fig 5

In order to develop a **Quasi-numerical Distinctive Syllogistic** QD, one must begin with the works of Carnes R.D. and Peterson P.L. (1991), who use the so-called “intermediate” quantifiers, which we have positioned on the vertical sides of the Quadrilateral, while the Distinctive arrangement favours, as do primitives, the Qn positioned on the upper horizontal side. An example of QD may be: (Only a majority part of the animals is ill)*(no ill animal is prized)→ (no, or only a minority part of animals is prized) (>ai * Eip) → (E < ap). If the infinite multiplications of the sides of the polygon are taken to the opposite extreme, we find the *borderline* case that reduces the figurations deal with up until now to simple segment (**collapsed polygon**): if b consists of only 1 element, it is either a or a’. This structure is analogous to that of the much discussed Singular Predications. In general, the validity of the 8 deductive formulas of modern **Singular Syllogisms** are confirmed (see Bird, O. 1964: Chapter 5, paragraph 45), while the U-proposition schemes will be added.

Compound Distinctive Syllogisms

As has been seen with the 3 categorical propositions of an oppositive triangle, the 7 topological situations in Figure 1A have been partitioned. Each of these divisions requires a further predication in order to be distinguished all from the others.

Polysyllogism with Complementary D7c

Basic expression, immediate inference, mediated deductive rules

The additional information required to distinguish the 7 cases may be provided by the Exclusive propositions. In Table 2a, we can see how the conjunction of pairs of predicates may do it by means of the two categorical and exclusive triangles (the blue one and the red one, with the latter being upside-down).

Tab2a

Xba ← **3** → ***Uba*** ← **5** → **Wba**
↑ ↑ ↑
1 **4** **7**
↓ ↓ ↓
Aba ← **2** → **Jba** ← **6** → ***Eba***

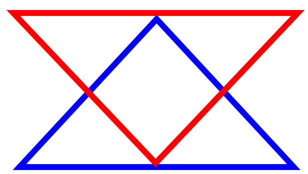


Table 2b below presents equivalent forms in each line. In column II ("scholastic"), the conjunctions of Categorical and Exclusive "triangular" propositions of the same pair ba are listed. These conjunctions identify the 7 topological cases indicated in column I on a one-to-one basis.

Table 2b

I cases	II "scholastic"	III explicit	IV practice	V alternative	VI synthetic
1	Aba *Xba	Aba *Ab'a'	AbaA,,	Aba *Eb'a	Abab'E
2	Aba *Jba	Aba *Yb'a'	AbaY,,	Aba *Yb'a	Abab'Y
3	Yba *Xba	Yba *Ab'a'	YbaA,,	Yba *Eb'a	Ybab'E
4	Yba *Jba	Yba *Yb'a'	YbaY,,	Yba *Yb'a	Ybab'Y
5	Yba *Wba	Yba' *Ab'a	Yba'A,,	Yba *Ab'a	Ybab'A
6	Eba *Jba	Aba' *Yb'a	Aba'Y,,	Eba *Yb'a	Ebab'Y
7	Eba *Wba	Aba' *Ab'a	Aba'A,,	Eba *Ab'a	Ebab'A

In column III (explicit), each "double" categorical proposition – or bi-categorical proposition - may become a premise of a compound distinctive syllogism or a **Distinctive Polysyllogism** with **Complementaries, D7c**, involving <u>3 terms</u> – of which one has middle function – as well as <u>3 corresponding complementary terms</u>. Column IV (practice), in light of *polysyllogistic* **calculation**, *simplifies the formula* of each case by omitting the conjunction sign and the second pair of terms, indicated here by the double comma (totally ignored during the calculation). We maintain the quantifier of the second pair, which is intended as the negated terms of the first.

First of all, the **rules** of **immediate inference** must be established (with reference to column IV, "practice"):

A..A inverts the sign of complementation of both terms. It is also immediately convertible.

A..Y or Y..A invert both the sign of complementation of each term and the succession of the quantifiers. The conversion of the terms must be accompanied by an analogous change in the position of the quantifiers. (e.g.: YabA = AbaY = Aa'b'Y = Yb'a'A)

Y..Y the sign of complementation may be inverted, even if of only one term. It is also immediately convertible. (ex.: YabY = YbaY = Yb'aY = Ya'b'Y…)
Table 3 illustrates the conclusions that may be extracted from 49 pairs of bi-premises with the exclusion of equivalent and subordinate moods. Conclusions are indicated by means of bi-categorical propositions, simple distinctive particulars, and traditional particulars, with conjunctions of bi-categoricals being excluded for purposes of conciseness.

	Table 3	1	2	3	4	5	6	7
		AbaA,,	**AbaY,,**	**YbaA,,**	**YbaY,,**	**Yba'A,,**	**Aba' Y,,**	**Aba'A,,**
1	**AacA,,**	**AbcA,,**	**AbcY,,**	**YbcA,,**	**YbcY,,**	**Ybc'A,,**	**Abc' Y,,**	**Abc'A,,**
2	**AacY,,**	**AbcY,,**	**AbcY,,**	***Ibc***	***Ycb***	**Ybc'A,,**	***Ib'c***	**Ybc'A,,**
3	**YacA,,**	**YbcA,,**	***Ib'c'***	**YbcA,,**	***Yc'b***	***Ibc'***	**Abc' Y,,**	**Abc' Y,,**
4	**YacY,,**	**YbcY,,**	***Yb'c***	***Ybc***		***Ybc***	***Yb'c***	**YbcY,,**
5	**Yac'A,,**	**Ybc'A,,**	***Ib'c***	**Ybc'A,,**	***Ycb***	***Ibc***	**AbcY,,**	**AbcY,,**
6	**Aac' Y,,**	**Abc' Y,,**	**Abc' Y,,**	***Ibc'***	***Yc'b***	**YbcA,,**	***Ib'c'***	**YbcA,,**
7	**Aac'A,,**	**Abc'A,,**	**Abc' Y,,**	**Ybc'A,,**	**YbcY,,**	**YbcA,,**	**AbcY,,**	**AbcA,,**

The **rules** of **practical deduction** will be applied as follows. Commas will be ignored and eventually replaced upon conclusion of the calculation if necessary. The <u>*standard form*</u> must be reached preliminarily: 1) the premises are arranged in sequence, as in the *first figure of the traditional syllogism with inverted premises* 2) the middle term is directed to the same sign in both premises (always possible through the application of the laws of immediate inference). Therefore:
A) When at least one of the premises has the form of <u>A..A</u>, the non-middle term contained therein may **substitute** the middle term in the other premise, thereby producing the conclusion.
B) If the premises have form <u>A..Y</u> or <u>Y..A</u>, and are equally "oriented" when in sequence (AY*AY, YA*YA), **the middle terms may be eliminated together with the nearby quantifiers**, leaving the conclusive sequence; if instead the "orientation" is symmetrical (AY*YA,YA*AY), the result will be a **I-conclusion** with terms of **the same sign as in the premise** if the central quantifiers (near the middle terms) are <u>A A</u>, and of the **opposite sign** if the central quantifiers are <u>Y Y</u>.
C) When a <u>Y..Y type premise</u> is combined with a <u>A..Y</u> or <u>Y..A</u> premise, the conclusion will be a **Y**-type whose **subject** will be the non-middle term that appears in the <u>A..Y</u> or <u>Y..A</u> premise and will have the **same sign as in the premise** if *accompanied by a Y* therein, and the **opposite sign** if *accompanied by a A* therein.
D) Two <u>Y..Y</u> premises result in **no conclusions** as all seven relations between non-middle terms are possible in this case.
The rules may also be given an interpretation in terms of set theory, for example: B) means: If the middle class is included (strictly speaking) in the other two, then the intersection between them is not empty; if the middle class includes the other two, the complementary of the latter will have an intersection that is not empty.
An example of polysyllogism D7c: **(YscA,, * YctA,,) → YstA,,**
only some **s**ound films are in **c**olour and every **m**ute film is in **b**lack and white;
Only some **c**olour films are by **T**ornatore, and every film in **b**lack and white is **n**ot by him;

therefore: only some **s**ound films are by **T**ornatore, and every **m**ute film is **n**ot by him;
Distinctive Polysyllogisms are more **economical** than their translation in equivalent traditional syllogisms with categorical propositions united by connectives.

Polysyllogisms with Subject "ad infinitum" D7s

Equivalent variations of D7c are possible. For example (see column V "alternative" of Table 2b), we can use two categorical propositions, of which the *second* presents only the **subject** of the first as **negative**, maintaining the *predicate unchanged* also using the universal negative quantifier; in a "synthetic" version (column VI), to not repeat the predicate "a", the second categorical proposition is read from right to left. According to this system of notation, we will obtain a syllogism equivalent to the previous one in the following form: **(Yscs'E * Yctc'E)→Ysts'E**
Only some **s**ound films are in **c**olour and no **m**ute film is;
only some **c**olour films are by **T**ornatore, and no **b**lack and white is;
therefore, only some **s**ound films are by **T**ornatore, and no **m**ute is;

Tri-Hexagonal Extensions of Classical Logics

Now Table 2b will be presented following its development with further interpretations (Table 2c).

I Cases	VI	III	IV	VII	VIII Tri-relational	IX iconic	X relations	XI synonymity antonymity
1	Abab'E	Aba *Ab'a'	AbaA,,	AbaA,,	bΘa	bΘa	**O**ccupies	**s**ynonym
2	Abab'Y	Aba *Yb'a'	AbaY,,	AbaY,,	b))a	b))a	**D**iminishes	hyp**o**nym
3	Ybab'E	Yba *Ab'a'	YbaA,,	Ab'a'Y,,	b'))a'	b((a	**C**lothes	hype**r**onym
4	Ybab'Y	Yba *Yb'a'	YbaY,,	YbaY,,	b)( )(a	b)( )(a	**M**ulti-mixes	**m**eronym
5	Ybab'A	Yba' *Ab'a	Yba'A,,	Ab'aY,,	b'))a	b( )a	**H**yperintegrates	hype**rc**omplement
6	Ebab'Y	Aba' *Yb'a	Aba'Y,,	Aba'Y,,	b))a'	b) (a	**K**eeps-out	hyp**oc**omplement
7	Ebab'A	Aba' *Ab'a	Aba'A,,	Aba'A,,	bΘa'	b ◡◠ a	**S**tops	**c**omplement

Iconic Relational Syllogism R7

If the predicative structure of the Distinctive Polysyllogism places it within the framework of a natural and traditional language, the translation of column IV, through the equivalences of column VII, to column VIII brings this system to belong to *Modern Symbolic Logic*, even if of a **limited** *kind*, in that is it **bi-argumentive** and **special** due to its type of **symbology** and the **reference** to the **7 cases**. In Column VII there are 7 cases described in a way "equivalent" to those "practice" column expressions already seen. Now a relational notation system may be adopted (column VIII), according to the following translation code (the dots stand for the variables): **A..A** becomes **Θ** ; **A..Y** becomes **))** ; **Y..Y** becomes **)( )(**. The IX column calls attention to its **diagrammatic iconicity** and is determined by means of the **immediate inference** rules of the previous column: the simple rule of <u>**specular**</u> rotation of the **parenthesis** or other **grapheme** in conjunction with the **inversion** of the quality of the nearby term [e.g. **b))a** = **b) (a'** = **b'((a'** = **b'( )a**]. The "Θ" relation consists of two parts (upper and lower) each of which can refer to a term: if only one part rotates around its extremity, it results in a ◡◠ or ◠◡ relation, if they *both rotate the "Θ" is restored*. Instead, the rotation of)( )(*are not affected by free changes in the quality of terms*. In column IX, a *miniature* scheme of the topological cases used as reference for each ordered pair of sets b a is graphically created. In column X, the 7 relations are denoted with transitive verbs.

The Oppositive Triangle in Propositional Logic

For the isomorphism that exists between *Class Logic* and **Propositional Logic**, we can export the tri-hexagonal scheme of distinctive syllogistics to the latter. By beginning with the structural similarities between the Universal Aba and the implication b → a, or between Aba' and b → a', we may obtain a new **distinctive implication** or **semi-implication** by analogy with the Particular Distinctive Proposition, which can be expressed as **b -:> a** and interpreted as "**b only partially** (or only sometimes) **implies a**" , equivalent by obversion to **b -:> a'**.

Obviously, even D7 or QD may inspire homologous structures in Propositional Logic. Certain interpretations may give the system ***temporal*** connotations (always, never, only sometimes, often, rarely…) or ***spatial*** connotations (everywhere, only in few places… in the same place, in the remaing space..).

The Oppositive Hexagon (and other Polygons) in Modal Logics

The tri-hexagonal structure is perhaps capable of simplifying modal systems. If, within a UD of "*considerable worlds or cases*", the **possible worlds or cases p** constitute a **set** *like any other* (complementary to that of the impossible, p'), we can express the relation with a *second set*, for example that of the *worlds* marked by the *c-characteristic*, as follows:

☐ Apc= every possible case is c = c is (Necessary) **Ascertained** or Evident or Agreed

■ Epc= no possible case is c = c is (Impossible) **Excluded** or Rejected

⬗ Ypc= only some possible cases are c = c is (Contingent) **Variable** Ambivalent Uneven Frayed Fuzzy Discussible Controversial

◆ Opc= at least 1 possible case is c' = c is (Unnecessary) **Excludable** Rejectable

◇ Ipc= at least 1 possible case is c = c is (*Possible*) **Ascertainable** Agreeable Likely

◨ Upc=either every or no possible case is c = c is (either Necessary or Impossible) **Prejudged** Predetermined Dogmatic Absolute Clean Crisp .

In this way, **alethic** expressions, *traced back to syllogistics*, become ***truth-functional***, as do the other modalities: **deontic** (b is Obligatory = b' is Forbidden; b is Forbidden = b' is Obligatory; b is Optional = b' is Optional) where the negation of Optional is Regulated, the negation of Forbidden is Permitted, and the negation of Obligatory may be "Gratuitous"; **epistemologic** (b is Demonstrable = b' is Falsifiable, b is Falsifiable = b' is Demonstrable, b is Undecidable = b' is Undecidable); **doxic** (believe, undervalue, doubt); **axiological** or **evaluative** (is good, is bad, is indifferent); **bulomatic**, (wish, etc). These modalities can be traced back to the tri-hexagonal scheme with the annexed inferential laws.

Epistemic modalities (know, ignore, insight/intuit, or deceive and so on) are instead *more problematic and complex*, because they involve both doxic and alethic modalities (perhaps even more); e.g. if I know b, b is ascertained and I believe it.

Even other polygonal schemes may be used. In fact, 5 **probabilistic** modality operators may be considered analogous to Quasi-numerical ones: **c**ertain (or **t**rue), **p**robable, **e**qui-probable, imp**r**obable, **i**mpossible (or **f**alse). Some possible disjunctions are: **v**erified=(c+i), **u**ncertain=(p+e+r), po**s**sible=(u+c). Intermediate operators are possible in many modalities, thereby creating a scalar order among them: e.g. "good enough" between good and indifferent, and so on. (See Horn L.1989).

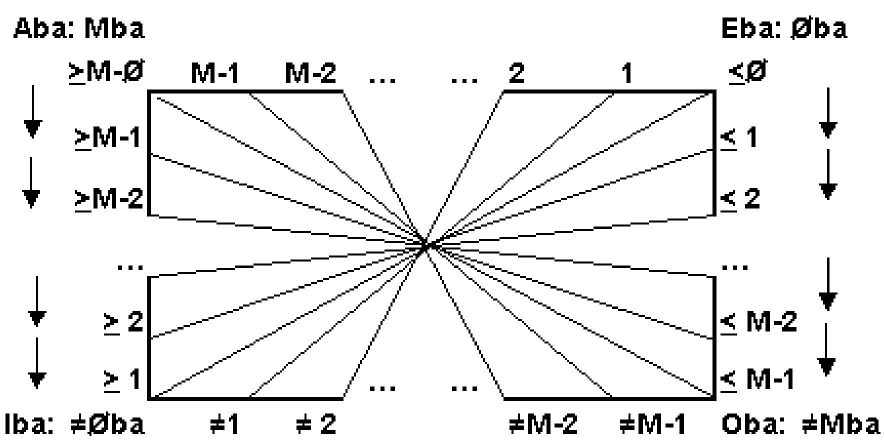
Other interpretations/applications

Column XI, table 2c offers an interpretation of the 7 relations in terms of **synonymities**; the first three and the last one are already used in **linguistics** and in *dictionaries of synonyms and antonyms*; the remaining three were coined by us to underline the importance of the *Conceptual Background*, or *Lexicographics*, in the definition of voices that plays a role analogous to that of the Universe of Discourse in the defining of the classes. In various disciplines (e.g. Semiotics), the Definitions may make use of the *Essential, Alien, Accidental* triad (their contradictories are Unessential, Compatible, Discriminant or Crucial). New **semiotic polygons** can be constructed.

Part II NUMERICAL DISTINCTIVE SYLLOGISMS N

Simple Numerical Distinctive Syllogisms ND **Numerical Distinctive Square** NDs

We define a *Numerical Distinctive Proposition*, a copulative relation between two, non-empty classes, a subject-class **s** and a predicate-class **p**, in a *UD* that is distinct from each of the two classes. This is a relation defined by means of **numbers** (normally Natural numbers) that make up the **Numerical Distinctive Quantifiers (NDq)** associated with the **functors**: **≥ (at least), ≤ (at most)**, > (more than), < (less than), ≠ (**either more or less**, or **not equal)**, **"=" (exactly** or **only)**; the latter *is implied*. By definition ≥ n is equal to > n - 1, as ≤ n is to < n+1. The NDq calls for a reference to ***three*** numbers for *each term*: the first (**Numerator N**) indicates the *quantity* or cardinality (of the subset of the subject) involved in the predication, the second indicates the quantity within which (**total** or **Maximum M**) the first *is distinguished*, and the third, *from which* (**Remainder R**), always the first, *is distinguished*; given that the remainder can be defined by the *difference* between total and numerator, which is usually **implied**. In practice, often when we speak of "*quantifiers*" or "*quantified*", we intend to refer only to the numerator, e.g."4 s are p"= s^4p.
All the types of traditional **quantifiers** may, in the final analysis, be **reducible** to the generalized **numerical form** of **sM^Np.** As we may easily assert, we obtain Asp when M=N, Esp when N= Ø, Ysp when (M ≠ N) * (N ≠ Ø), Isp when N ≠ Ø, Osp when M ≠ N, Usp when (M=N aut N=Ø). Even the **QD** may be reduced to the numerical schemes, with numerical variables subject to conditions: when **N >½M** we may affirm that **>sp**, when **N =½M**, then **!!sp**, when **N < ½M** , **<sp**. On these basis we constuct simple **Numerical Distinctive Syllogism ND**: [($b_6{}^4$ **r**)*($r_9{}^{\varnothing}$ **c**)] → ($b_6{}^{\leq 2}$ **c**) (of all our **6 b**icycles, only **4** are **r**usty) and (**none** of our **9 r**usty objects is **c**ostly), therefore: (**at most 2** of our **6 b**icycles are **c**ostly). As for the Quasi-numerical propositions, we can use Oppositive Polygons even for the Numerical Distinctive Propositions; however, in order to **integrate** the consequential ND with other more Classic Numerical Syllogisms NS, we modify the Oppositive Polygon to adapt it to a Numerical Distinctive square NDs. We thereby obtain a representation of the intermediate quantifiers in an **orderly scale** between the two ends of segment Aba-Eba. (Figure 6 below: the dots leave the expansion to other intermediate quantifiers open). Fig. 6:



Each quantifier is diametrically opposed to its contradictory. The quantifiers may be interpreted in an **Exceptive** sense: All except (or all but)Ø (zero), except 1, 2 and so on to "all except all", i.e. none. A long tradition, from the scholastic "excepta" to Lambert J.H., to De Morgan A., to the numerical propositions of Murphree W. (1997), had made use of the numerical exceptive, which can be compared to the set operation of asymmetric difference: **s\p** (or **s – p**) = the **s 's that are not p** (= the **complement of p *in s***). Each quantifier on the upper side (A-E) is arranged symmetrically to the median vertical line in correspondence to its own reciprocal; this may be defined by the equivalence **sM ^{N} p ←→ sM $^{M-N}$ p'**. If N is equal to M (or to Ø), the equivalence above would become **sM ^{M} p ←→ sM ^{Ø} p'** (or with numeral M and Ø switched). If s has an *even* number of elements, a numerator will be placed exactly on the median axis and will be its own reciprocal. The interval ***between*** **M-1** and **1** illustrates the numerators that, when disjointed, compose **Yba**, while the complementary - or ***external*** - interval indicates **Uba**. Other types of internal or external intervals may be obtained from the schemes. Therefore, s_6 $^{\geq 3 \leq 5}$ p , will be read: *Among the s 's, which number 6 in all, between 3 and 5 are among the p 's, which number at least 7 in all***,** while **s_6** $^{\leq 3 \geq 5}$ p will mean: *Among the s 's, which number 6 in all, either at the most 3 or at least 5 are among the p 's.*

The non-Distinctive "Numerical Syllogisms" by Hacker E.A. and Parry W.T. (1967), by Murphree W. (1997) and others, **ignore the upper side** of the NDs. However, their deductive apparatus appears to be sound and founded on algebraic developments and diagrammatical tests. The **deductive rules** of these systems must be referred to in order to construct more economical ND that maintain immediate intuition and the **principles of discrete mathematics** (inclusion-exclusion, Da Silva/Sylvester's formulas…). Below are two example of NS in the terms of the above mentioned authors, using our symbology to give an idea of a potential fusion.

(m $^{>60}$ p) * (m $^{\leq 10}$ s') → (s $^{>50}$ p) (numerical Disamis)

(m $^{>16}$ p') * (m $^{\leq 10}$ s') → (s $^{> 6}$ p') (numerical Bocardo)

As we know, the *HydraLynx* mode, typical of the Triangular Syllogism, may be derived from classic Disamis and Bocardo moods, therefore the examples given above give origins to an analogous mood in Numerical Distinctive Syllogistics.

($^{>16}$ m $^{>60}$ p) * (m $^{\leq 10}$ s') → ($^{> 6}$ s $^{>50}$ p) (numerical HydraLynx)

Numerical Distinctive Polysyllogisms NDP

The "distinctive spirit" naturally leads to a "numerical **quantification** of the **predicate**" and, in order to have full information, to a numerical quantification of the **complement of the subject**, or of the intersection of the complements of the subject and the predicate (bi-proposition). For example, $^{(2)}$ **s8 ^{6} 9p** means: "While exactly **(2) remainders are not** (are non-p) **of 8s totals, exactly 6 are among the 9p totals"**. The general formula of a **complete numerical polysyllogistic** proposition (also with a subject "ad infinitum") will be:

s…$_M$ $^{…Q…R}$ $_{…G}$ p $^{…T…L}$ $_{…V}$ s' (the dots count as occurrences of the functors ≤ or ≥) or, by ignoring functors, intervals and derivable data: **s**M ^{R} G**p** Z**s'.** According to these bases we obtain **Numerical Distinctive Polysyllogisms NDP**.

Regarding immediate inferences, from the summed total of the subject and that of its complement (but the same can be said for the predicate), we obtain the total of the UD. The difference between total and numerator indicate the power of the intersection between the subject and the complement of the predicate. From the total of the subject and predicate, minus the intersection, in order to avoid counting twice, we obtain the total of the union set. The UD minus this union equals the total of the complement of the union. If zeros appear, some sectors will be absent.

One possible numerical development of the iconic relational system may lead to a simplification. In fact, we may attribute a number to each of the sectors separated by the parentheses or other icons. For example:b8Θa 5 = b's and a's are equivalent and 8 at all, the remaining are 5

b8)3)a 6 = b 's are 8, a 's are 11, the not-b 's not-a's are 6.

6 b(3(8a = the not-b's not-a's are 6, the b's are 11, the a's are 8,

b5)(3)(7a 9 = the b's are 8, the a's are 10, not-a's not-b's are 9 [or b(5(3)7)a 9]

b 5(3)7 a = the b's are 8, the a's are 10, the not-a's are 5, the not-b's are 7

b8) 2 (11a = the b's are 8, the a's are 11, 2 are neither a nor b

b4 ∪∩ 3a = the b's are 4, the a 's are 3

Part III NON-STANDARD LOGICAL INTERPRETATIONS
The Transposition of Quantifiers to Truth Values: Inter-bivalence, Super-bivalence

At the basis of standard logic, the **Principle of Non-Contradiction Pnc**, may be expressed faithfully in Singular propositions and syllogisms where a specific attribute may be entirely attributed or entirely negated to a subject that is "non-quantified" because it is unitary. Aristotle emphasised that "the same attribute cannot contemporaneously belong and not belong to the same subject ***from the same point of view***" (or aspect or interpretation). Let's then ask: and if we changed point of view? Multiple points of view are introduced with the quantification of the subject, to meet the need to **distinguish** partitions or subsets within the subject-class, which is no longer seen as a monolithic entity. In the Particular Distinctive Proposition, the subject-class (logic level 2), which is formally unitary, essentially consists of two subclasses (logic level 1) that are disjointed and exhaustive in relation to the mother-class, for which opposite propositions are defined that are, however, in conformity with their elements (logic level 0). In classical logic Pnc is associated with bivalence. "Are Europeans Greek?" As a rule, or generally speaking, they are not. But if what is true for **one part** is not true for the **other part** of the subject (level 1), how we can maintain that for the **whole** subject (level 2), **only one** of the two truth-values is true? A strict bivalence seems *inadequate* because it is **asymmetrical.** We know the properties of a class may be different to those of its own elements: why can this not be true for the kind of truth-values as well?
We can nullify this inadequacy by *weakening the Principle of Contradiction to level 2*, or in other words, partially violating it by means of a **flexible bivalence.** There will then be three possible and alternative answers to the question like "Are x's y's?": **T** (True) = yes, it is True; **F** = no, it is False; **Γ** =so-so, **partially,** it is **Limited** (or Partial, Tolerant, Reductive, Moderate, Relative). If a proposition is **partially true**, it is also **partially false**, therefore **partially not false** and **partially not true.** The tripartition **each/none/only some** then shifts from the **quantifier to the truth value**, or even to the **copula is/is not/is partially**, if preferred. It should be noted that the "third" value option is ***nothing but* a hybrid** consisting exclusively of "portions" of the other two, and therefore does not result as being "alien" or "heterogeneous", as required by an authentic third value, extra-bivalent (lacking in meaning, not well-formed, conventional, non-contextual, extra-categorical, inaccurate: see the oppositive hexagon of Sinowjew A.-Wessel H. (1976), that is trivalent on a level of terms, bivalent on a propositional level). This valence instead refers to all those cases ideally positioned in the interval between the two extremes (excluded) of "Every" and of "None". For this reason we have denoted it as **Inter-bivalence** (or **Meso-bivalence**), and ***not Trivalence***. Therefore it is necessary to make a conceptual distinction between the two formulations that have always been believed to be equals: on one hand the Aristotelian principle of the μεταξυ or intermediate (**Excluded Middle**), and, on the other, its scholastic Latin version of "tertium non datur" (**Excluded Third**). Abandoning the classic bivalence to make room for the Inter-bivalence, is due **not** to modal, temporal, metaphysical, or physical/probabilistic needs (free will, indeterminism of future), as occurred historically for Aristotle, Lukasiewicz, and others, but to ***multiple perspectives*** and a ***stratified*** *way of observing* the ***semantic-topological relations*** between sets, subsets, and elements.
There is also a stronger way of *remaining within the bivalence by violating the Pnc*: that of **Paraconsistent Logics** or Dialectics, according to which something can be **both true and false** (**Ŧ**). We maintain that such Logics, that proponents themselves would save explicitly from trivial inconsistency, are **either** implicitly **Tri-Polivalent**, **or** in some way **related to Inter-bivalence**. In this case, **the logic levels themselves (0, 1, 2,…)** will be **translated** in terms of **valence**, **not the quantifier**, with a "metalogical" lateral leap. E.g.: all machines are useful (level 1A: if well used); all machines are harmful (level 1B: if badly used); thus all machines are both useful and harmful (level 2: without considering their use). **Only some levels** confirm them as being useful. It is true and false that machines are useful. We can define this valence (**Ŧ**) as a **Super-bivalence**. Many

thinking systems adopt it, e.g: Imaginary logic of Vasiliev N. (1925), Non-Monotonic Logics, Psychoanalysis (contrariety conscious-unconscious, bi-logic of Matte-Blanco I.) and, in general, the non-reductionist descriptions of reality, from biology to historical and sociological processes. The “Super-bivalence” may interpret contradictory sets (and resolve the liar paradox) in that they are unfolded in overlapping dimensions or perspectives (spatial-temporal, topological, polysemy, metalinguistic, structural, etc.) with **synthetic judgments oscillating between the various planes**. We can conclude that **isomorphisms** between the *bivalent D-systems* and many ***Non-Standard Logics*** may be identified in the following way: by **depriving** the subject class of its **quantifier** (or of its **logic level**) and **transposing** its (pre)numerical attribute to the **truth value** of the judgment. Their typical law is obversion: Yba=Yba’; **Γba = Γba’ ; Ŧba = Ŧba’ .**

Pre-numerical Inter-bivalent Syllogisms (generalizing)

We can thereby produce bi-propositions or polysyllogisms in Inter-bivalent terms as follows:

1 bΘa Abab’E T bab’ F It is true that "b is a" and that "b’ is a" is false
2 b))a Abab’Y T bab’ Γ It is true that "b is a" and that "b’ is a" is limited
3 b((a Ybab'E Γ bab' F It is limited that "b is a" and that "b’ is a" is false
4 b)( )(a Ybab’Y T bab’ Γ It is limited that "b is a" and that "b’ is a" is limited
5 b()a Ybab'A Γ bab' T It is limited that "b is a" and that "b’ is a" is true
6 b)(a Ebab'Y F bab' Γ It is false that "b is a" and that "b’ is a" is limited
7 b◡◠a Ebab’A F bab’ T It is false that "b is a" and that "b’ is a" is true

That “men are mortals” *is true*, vice-versa, that “mortals are men” *is true in part*;
That "mortals are birds" *is true in part*, vice-versa, that “birds are mortal” *is true*;
therefore, "non-men are non-birds" is *at least partially true*.

The I-quantifier will translate as “at least in part”, the O-quantifier will translate as “at most in part”, the U-quantifier will translate as “totally or not at all”. For Qn the translation may be analogous.

Numerical Inter-bivalent Syllogisms (punctualizing)

These assign a precise **measure of truth** of a subject-class predication towards a predicate-class. In reference to the numerical square, we can make a truth value correspond to a point or to an interval defined by points on the segment between the universal quantifiers. Naturally, the measure of **truth** is **relative** to the **totality** of the subject-class (usable in various ways: absolute commensurable, correlative-statistical, relative-probabilistic, fractionary-percentages, etc). Here, as in the following systems, all the deductive rules and laws for numerical syllogisms can be found. The Inter-bivalence must be *distinguished from the probability calculus*, even if it uses an analogous mathematical method. The probabilistic uncertainty disappears when the event occurs, while the partial truth does not. The 50% uncertainty that the refrigerator contains an apple is all but the certainty that there is half an apple (Kosko B.1993). We can arrive at infinite numerical quantifiers, like **fractions** between 1 e Ø, or **percentages**. Attention must be paid, however, to reductive interpretations: e.g. the NDq in $s10^{5}$, if interpreted as half of s’s (because arithmetically ½ = 5/10), leads to the loss of the absolute data of s, leaving us with only some relations that may be important in some contexts, but misleading in others.

Scalar or Graduated (discrete) Numerical Inter-bivalent Syllogisms

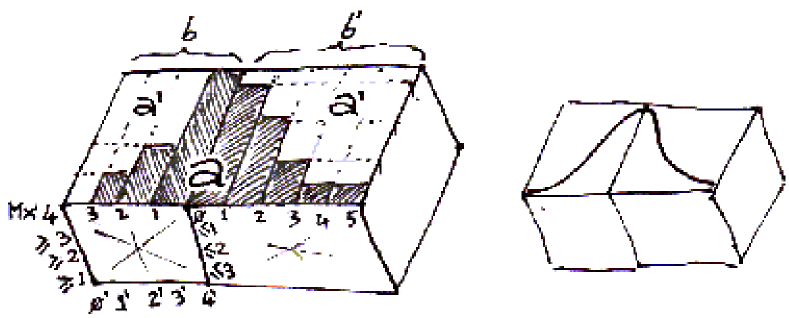
The **Natural Numerical Syllogisms** seen previously are **translated** here in a sort of *Poli-Inter-bivalence*. By using a metaphor, if the bivalence uses black and white and the Inter-bivalence adds a grey tone, the Scalar logic **increases** the **intermediate** tones according to **discrete** unities organized in a **scale**, thereby ensuring a more **precise** judgment. It is a first approximation of the fuzzy propositions, between term classes from confines that are still crisp (rough), where the belonging or non-belonging of any element to the given set may still be clearly defined. One of the results of this logic is that it can solve the paradoxes of sorite (or heap).

Fuzzy (continuous) Inter-bivalent Syllogisms

With the use of quantifiers with **rational or real** numbers, the **infinite** range of shades of grey is **reached** without an interruption: **Fuzzy Syllogisms and Polysyllogisms** (flou, nuance, soft), however, **referred to the relation** or predication **between sets** (level 1 or 2) that remain **crisp**. The interpretation opens towards **continuous entities**, for example surfaces, distances, etc., maintaining the reference, however, to the natural language guaranteed by the predicative structure. The imaginary factor of a number may define the predication in modal or temporal terms.

Three-dimensional Fuzzy Syllogisms and Polysyllogisms

In order to complete this approach to fuzzy logic, there is still one last step: the attribution of the **measure (or truth) of its membership** to a class (thereby transforming it into a *fuzzy class*) as well as to **each element** (level 0) of the class itself and *not only to the relation*. This may be expressed by adding a **third cartesian axis** to the model of the square, which becomes a "**Fuzzy Cube of Opposition**" (see Figure 7 below).



If possible, it will then be necessary to identify the mathematical functions that describe the progression of orderly individual values (membership function).
According to the Fuzzy Cube and the rules acquired through this Inter-bivalence, it will then be possible to develop **Three-dimensional Fuzzy Syllogisms and Polysyllogisms**.

Potential for theoretical development

The **limitations** of D systems derive from being founded on the **subject-predicate structure** (term logic), which is not suitable for expressing complex, multi-argument functions and relational logic. However, the analyses of the distinctive propositions reveal a relation with four-five arguments (which may be even more numerous in polysyllogisms) in which the first two (subsets) share the third (subject) and relate to a fourth and a fifth argument (the predicate and its negation). We then saw a biargument relational interpretation in R7, which is however **numerically** quantifiable. In the historical development of classical logic, the Exceptive Proposition ["Every b, except c, is a ", *A(b-c)a*] manifests a tri-argument structure. In *Lambert's* studies (1764), expressions like "*all b's that are c's are not a's*" (which may be expanded iteratively to other terms: "that are…that are…"), are similar to *Boolean functors*. All this suggests that an **extension** of the argumental and relational variables of syllogistics **is possible** without having to give up natural language and intuition. **Summers F.** (1990) created a system (called "*Old New Logic*") that began with the subject-predicate structure as a base of the natural and syllogistic language and underwent progressive expansion (studies by **Englebretsen G.** (1987) and **Numerical Term Logic** by **Murphree W.** (1998)). This system seems to have a deductive and expressive power that rivals **first order predicate** (and **relation**) **calculus**. According to **Lindell S.** (2005), these alternatives may actually be connected (and applicable in information technology and A. I.). Recently **Ben Yami H.** (2004) has constructed an other deductive system based on natural language, comparable in its power to the first order predicate calculus. May be an integration of some of these systems is possible.

Potential applications

The history of science has amply demonstrated how pure theories and applied technologies favour reciprocal development if they interact and contaminate each other. We support a develop of a common syntax between *Classic, Fuzzy and D-Logic*. Vast areas of disciplines apply only classical logics, while others are strangers to any type of logic. The application of natural language systems, like D, is proposed with a view to clarifying terminology and transmitting knowledge in fields where *plurality of codes* and neologisms reign and are today growing exponentially. In Fig 8, every letter-sign can be predicated in its being "H", below a degree of truth, complementary to that of its "A"-predicability. A fuzzy syllogistic can well conceives the sequence of intermediate values for similar problems of "pattern recognizing", where bivalent models fail because the difficulty to interpret the ambiguity of the cases. Fig. 8:

As an analogy and as integration of fuzzy technologies (especially in household appliances and regulation and control systems), applications may be considered in *Engineering* (**machine design**, e.g. Donnarumma A. and Pappalardo M.,1999), *Biomedicine* (**compared analyses**), *Linguistics* (**translation software, frequentistic dictionaries**), *Library Science* (**indexing**), *Information Technology* (**search engines**), *Law* (**comparative law**), *Multimedia Communications* (measuring the efficiency of messages, didactics).

Montreux (CH), May 31 – June 3, 2007

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Appen1a	Ama	Am'a'	Ema	Em'a'	Ima	Im'a'	Oma	Om'a'
Amc	Ica	Ac'a'	Oca	Ec'a'	Ica		Oca	
Am'c'	Aca	Ic'a'	Eca	Oc'a'		Ic'a'		Oc'a'
Emc	Oc'a'	Eca	Ic'a'	Aca	Oc'a'		Ic'a'	
Em'c'	Ec'a'	Oca	Ac'a'	Ica		Oca		Ica
Imc	Ica		Oca					
Im'c'		Ic'a'		Oc'a'				
Omc	Oc'a'		Ic'a'					
Om'c'		Oca		Ica				
Ymc	**Yac**		**Ya'c'**					
Ym'c'		**Ya'c'**		**Yac**				
Ycm	**Ica**	**Oca**	**Oca**	**Ica**				
Yc'm'	**Oc'a'**	**Ic'a'**	**Ic'a'**	**Oc'a'**				
Umc		**Uac**		**Ua'c'**				
Um'c'	**Ua'c'**		**Uac**					
Ucm	Aca+Oc'a'	Eca+Ic'a'	Eca+Ic'a'	Aca+Oc'a'	Aca+Oc'a'	Eca+Ic'a'	Eca+Ic'a'	Aca+Oc'a'
Uc'm'	Ica+Ec'a'	Oca+Ac'a'	Oca+Ac'a'	Ica+Ec'a'	Ica+Ec'a'	Oca+Ac'a'	Oca+Ac'a'	Ica+Ec'a'

Appen1b	Yma	Ym'a'	Yam	Ya'm'	Uma	Um'a'	Uam	Ua'm'
Amc	**Yca**		**Ica**	**Oca**		**Uc'a'**	Oca+Ac'a'	Ica+Ec'a'
Am'c'		**Yc'a'**	**Oc'a'**	**Ic'a'**	**Uca**		Eca+Ic'a'	Aca+Oc'a'
Emc	**Yc'a'**		**Oc'a'**	**Ic'a'**		**Uca**	Eca+Ic'a'	Aca+Oc'a'
Em'c'		**Yca**	**Ica**	**Oca**	**Uc'a'**		Oca+Ac'a'	Ica+Ec'a'
Imc							Oca+Ac'a'	Ica+Ec'a'
Im'c'							Eca+Ic'a'	Aca+Oc'a'
Omc							Eca+Ic'a'	Aca+Oc'a'
Om'c'							Oca+Ac'a'	Ica+Ec'a'
Ymc					Yca+Yc'a'			
Ym'c'						Yca+Yc'a'		
Ycm							**Oca**	**Ica**
Yc'm'							**Ic'a'**	**Oc'a'**
Umc	Yca+Yc'a'					Uca+Uc'a'		
Um'c'		Yca+Yc'a'			Uca+Uc'a'			
Ucm			**Oc'a'**	**Ic'a'**			Eca+Ic'a'	Aca+Oc'a'
Uc'm'			**Ica**	**Oca**			Oca+Ac'a'	Ica+Ec'a'

Appendix 2: Quantification of the Predicate as Polysyllogism with Inverted Pairs of Terms (D10)

In psycholinguistics, the study of the logical structure of natural languages would lead to the conclusion that W. Hamilton's 19th-century "**Quantification of the Predicate**" and similar systems may correspond better to the economy of human thought (see Seuren, P.A.M. 2006) than the modern predicate calculus. Similar systems may be obtained from a **polysyllogism D10** similar to D7, slightly more limited and redundant (10 cases). In these bi-propositions, the second quantifier is referred to the predicate (see 1st Table below, last column) or, postponed, reading from right to left, as referred to the same **pair of classes** in **inverted order**: For example: Aba * Yab = AbaY = AbYa = all b 's are only some a 's. The 2nd Table compares the basic expressions of D10 to which used by some supporters of predicate quantification in the past centuries. (Note: Gergonne was not a supporter, but the "spirit" of his system is similar to D10).

7Cases	**Quantification of Inverted Pairs**		Diagrams – Quantification of the predicate		
1	AbaA <--> Ab'a'A	---> Eba'E ---> Eb'aE	Θ		AbAa = Aba * Aab
2	AbaY <--> Yb'a'A	---> Yb'aY ---> Eba'E	))		AbYa = Aba * Yab
3	YbaA <--> Ab'a' Y	---> Yba' Y ---> Eb'aE	((		YbAa = Yba * Aab
4+2	Yb'aY		)) or	)( )(	Yb' Ya = Yb'a * Yab'
4+3	Yba' Y		((or	)( )(	YbYa' = Yba' * Ya'b
4+5	YbaY		( ) or	)( )(	YbYa = Yba * Yab
4+6	Yb'a' Y		) (or	)( )(	Yb' Ya' = Yb'a' * Ya'b'
5	Yba'A <--> Ab'aY	---> YbaY ---> Eb'a'E	( )		YbAa' = Yba' * Aa'b
6	Aba' Y <--> Yb'aA	---> Yb'a' Y ---> EbaE	) (		AbYa' = Aba' * Ya'b
7	Aba'A <--> Ab'aA	---> EbaE ---> Eb'a'E	◡⌒		AbAa' = Aba' * Aa'b

7 CASES	D10	von Holland G.J.	Stanhope C.	Bentham G.	Hamilton W.	Gergonne J.D.
1	AbaA	**b/1=a/1 , b/∞=a/∞**	*all b = all a*	**Tb=Ta**	**all b is all a**	**b I a**
2	AbaY	**b/1=a/f**	**all b = some a**	**Tb=Pa**	**all b is some a**	**b (a**
3	YbaA	**b/f=a/1**	some b = all a	**Pb=Ta**	**some b is all a**	**b) a**
4+2	Yb'aY	(b/∞)/f=a/g	some not-b = some a	*Pb'=Pa*	*some b' is some a*	*b' X a*
4+3	Yba' Y	b/f=(a/∞)/g	**some b = some not-a**	*Pb=Pa'*	*some b is some a'*	*b X a'*
4+5	YbaY	**b/f=a/g**	**some b = some a**	**Pb=Pa**	**some b is some a**	**b X a**
4+6	Yb'a' Y	(b/∞)/f=(a/∞)/g	some not-b = some not-a	*Pb'=Pa'*	*some b' is some a'*	*b' X a'*
5	Yba' Y	**b/f=a/∞ , b/∞=a/f**	some b = all not-a	*Pb=Ta'*	*some b is all a'*	*b) a'*
6	Aba' Y	b/1=(a/∞)/f , (b/∞)/f=a/1	**all b = some not-a**	*Tb=Pa'*	*all b is some a'*	*b (a'*
7	Aba'A	**b/1=a/∞ , b/∞=a/1**	*all b = all not-a*	*Tb=Ta'*	*all b is all a'*	*b I a'*

Plausible expressions of completion

Expressions of completion unknown to the author

Historical author's own expressions

Enriched by negative classes